CALCULATION OF THE CHARACTERISTICS OF A GAS-DYNAMIC LASER

N. A. Generalov, G. I. Kozlov, and I. K. Selezneva

UDC 533.6.0.11.8

The dependence of the radiated power on the characteristics of optical cavities in the case of flow systems has been investigated in a number of papers [1-3], in which it is assumed that population inversion of the laser levels is obtained until entry into the cavity. The operation of a cavity is analyzed in [1] in the geometric-optical approximation with allowance for vibrational relaxation in the gas flow. A simplified system of relaxation equations is solved under steady-state lasing conditions and an expression derived for the laser output power on the assumption of constant temperature, density, and flow speed. The vibrational relaxation processes in the cavity itself are ignored in [2, 3]. It is shown in those studies that the solution has a singularity at the cavity input within the context of the model used. In the present article the performance characteristics of a CO_2-N_2-He gas-dynamic laser with a plane cavity are calculated. A set of equations describing the processes in the cavity is analyzed and solved numerically. Population inversion of the CO_2 laser levels is created by pre-expansion of the given mixture through a flat hyperbolic nozzle. The dependence of the output power on the reflectivities of the mirrors, the cavity length, the pressure, and the composition of the active gas medium is determined.

1. Consider an adiabatic one-dimensional flow of a CO_2-N_2-He mixture through a flat hyperbolic nozzle having a special configuration such that it terminates in a plane-parallel channel. The mixture, expanded through the nozzle, enters the interior of a Fabry-Perot cavity.

It may be inferred on the basis of our calculations [4] of the population inversion of CO_2 molecules in expanded gas flows for an axisymmetrical nozzle that the main processes governing collisional relaxation in the CO_2-N_2-He mixture are the following:

$$CO_2(00^01) + M \underset{K_{-1}}{\overset{K_1}{\rightleftarrows}} CO_2(11^10) + M + \Delta\varepsilon_1 \tag{1.1}$$

$$CO_2(00^01) + N_2(0) \underset{K_{-2}}{\overset{K_2}{\rightleftarrows}} CO_2(00^00) + N_2(1) + \Delta\varepsilon_2 \tag{1.2}$$

$$CO_2(01^10) + M \underset{K_{-3}}{\overset{K_3}{\rightleftarrows}} CO_2(00^00) + M + \Delta\varepsilon_3 \tag{1.3}$$

Here M is any one of the CO_2, N_2, or He particles, $\Delta\varepsilon_m$ is the heat of the m-th reaction (m = 1, 2, 3), $K_m = \sum_n P_m^{(n)} Z^{(n)} \alpha^{(n)}$, $P_m^{(n)}$ is the probability of the deactivation or exchange of lower-level quanta in the m-th reaction in one collision of a CO_2 molecule with the n-th particle, $Z^{(n)}$ is the collision frequency of CO_2

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 33-40, September-October, 1972. Original article submitted May 10, 1972.

molecules with molecules of species n per unit concentration, and $\alpha^{(n)} = N^{(n)}/N$ is the molar fraction of the n-th component of the mixture. The subscripts n = 1, 2, 3 refer to the respective molecules CO_2, N_2, and He.

We follow the procedure developed in [5], i.e., assume that both during expansion and in the cavity local thermodynamic equilibrium exists within the vibrational degrees of freedom of the CO_2 and N_2 molecules, so that we can associate with each vibrational mode a vibrational temperature T_i (i = 1, .., 4). The values of i = 1, 2, 3 refer to the three modes of CO_2, and i = 4 to the vibrational motion of the N_2 molecule. It may be assumed with acceptable error that the vibrational temperatures T_1 and T_2 are equal.

The mass, momentum, and energy conservation equations, the equations of state, and the relaxation equations for one-dimensional steady-state flow of an ideal gas up to entry into the cavity have the form

$$A(x)\,\rho u = A_*\rho_* u_* \tag{1.4}$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = 0 \tag{1.5}$$

$$\frac{d}{dx}\left[\frac{5}{2}kT + (\alpha^{(1)} + \alpha^{(2)})\,kT + \alpha^{(1)}\sum_1^3 E_i + \alpha^{(2)}E_4 + \frac{1}{2}mu^2\right] = 0 \tag{1.6}$$

$$P = \rho m^{-1} kT \tag{1.7}$$

$$\frac{dy_2}{dx} = \frac{\rho}{mu}\,\frac{(1+y_2)^2(1-y_2)}{1+4y_2+y_2^2}\left\{-\frac{1}{2}K_3[y_2 - \exp(-\Delta\varepsilon_3/kT)] + \frac{3}{2}K_1\frac{y_3 - y_2^3\exp(-\Delta\varepsilon_1/kT)}{(1-y_2)(1+y_2)(1-y_3)}\right\} \tag{1.8}$$

$$\frac{dy_3}{dx} = \frac{\rho}{mu}(1-y_3)\left[-K_2\frac{y_3 - y_4\exp(-\Delta\varepsilon_2/kT)}{(1-y_4)} - K_1\frac{y_3 - y_2^3\exp(-\Delta\varepsilon_1/kT)}{(1-y_2)^2(1+y_2)}\right] \tag{1.9}$$

$$\frac{dy_4}{dx} = \frac{\rho}{mu}(1-y_4)\,K_2\frac{\alpha^{(1)}}{\alpha^{(2)}}\,\frac{y_3 - y_4\exp(-\Delta\varepsilon_2/kT)}{(1-y_3)} \tag{1.10}$$

Here ρ, u, T, p, and A are the density, velocity, temperature, pressure, and cross-sectional area of the nozzle, $m = \sum_n \alpha^{(n)} m_n$, m_n is the mass of the n-th species of molecule, E_i is the vibrational energy of the i-th vibrational degree of freedom per gas particle, and k is the Boltzmann constant. The values of the variables in the critical section of the nozzle are indicated by an asterisk.

The relaxation equations (1.8)-(1.10) are written in the variables $y_i = \exp(-h\nu_i/kT_i)$, where $h\nu_i$ is the vibrational quantum energy of the i-th mode. We have assumed in writing Eqs. (1.8)-(1.10) that the density of CO_2 molecules in a vibrational state (m, n, p) is described by the conventional relation given in [6]:

$$N^{(1)}(m, n, p) = N^{(1)}[y_1^m(1-y_1)][(n+1)\,y_2^n(1-y_2)^2][y_3^p(1-y_3)] \tag{1.11}$$

and relaxation of the vibrational temperatues is realized only through collisions. Spontaneous transitions can be neglected, because the radiation lifetimes of the levels far exceed the characteristic transit times of the gas through the system.

The energy of the vibrational degrees of freedom have the following form for a hypothetical harmonic oscillator model:

$$E_i = \frac{h\nu_i y_i}{1-y_i} \quad (i = 1, 3, 4), \qquad E_2 = \frac{2h\nu_2 y_2}{1-y_2} \tag{1.12}$$

The probabilities of the processes (1.1)-(1.3) are evaluated the same as in [4].

2. The mixture, expanded through the nozzle, enters a plane-parallel cavity, whose z axis is perpendicular to the direction of gas flow. The mirrors of the cavity are located at the points z = 0 and z = L. The reflectivities of the mirrors are equal to

$$r_1 = 1 - a_1 - t_1, \quad r_2 = 1 - a_2 - t_2 \tag{2.1}$$

where a_1 and a_2 are the loss coefficients due to energy dissipation and t_1 and t_2 are the transmissivities of the first and second mirrors, respectively.

For a steady flow of gas through the cavity the conservation equations for the mass (1.4) and momentum (1.5) and the equation of state (1.7) remain the same as during expansion through the nozzle. With allowance for the emission of radiation and dissipation at the mirrors the energy equation assumes the form

$$\frac{\rho u}{m}\frac{d}{dx}\left[\frac{5}{2}kT + (\alpha^{(1)} + \alpha^{(2)})\,kT + \alpha^{(1)}\sum_1^3 E_i + \alpha^{(2)}E_4 + \frac{1}{2}mu^2\right] = G \tag{2.2}$$

Here G is the energy lost per cubic centimeter of gas per second. Under steady-state lasing conditions,

$$G = \int K_\nu I_\nu d\nu \tag{2.3}$$

where K_ν is the optical gain of the medium and I_ν is the spectral intensity of the emitted radiation. Since the gain scarcely changes over the width of the laser line, the expression for G is written in the form

$$G = K_{\nu_0} I \tag{2.4}$$

where $K_{\nu 0}$ is the gain at the center of the line and I is the total radiation intensity.

The relaxation equations have the following form in the cavity with allowance for stimulated transitions:

$$\frac{dy_2}{dx} = \varphi_1 + \frac{1}{u}\frac{(1+y_2)^2(1-y_2)^2}{1+4y_2+y_2^2}\frac{K_{\nu_0} I}{h\nu N^{(1)}} \tag{2.5}$$

$$\frac{dy_3}{dx} = \varphi_2 - \frac{1}{u}(1-y_3)^2\frac{K_{\nu_0} I}{h\nu N^{(1)}} \tag{2.6}$$

$$\frac{dy_4}{dx} = \varphi_3 \tag{2.7}$$

Here the functions φ_1, φ_2, and φ_3 represent the right-hand sides of the respective equations (1.8), (1.9), and (1.10).

To close the system (1.4), (1.5), (1.7), (2.2), (2.5)-(2.7) we require one more equation. The required equation is given by the steady-lasing condition obtained in [1]:

$$2LK_{\nu_0} = -\ln(r_1 r_2) \tag{2.8}$$

which is a consequence of the equality of the radiation loss and gain in two-way transit through the cavity.

3. The optical gain K_ν of the medium is defined as follows, according to [7]:

$$K_\nu = \frac{1}{I^+}\frac{\partial I^+}{\partial z} = -\frac{1}{I^-}\frac{\partial I^-}{\partial z} \tag{3.1}$$

where I^+ and I^- are the light flux intensities in the positive and negative z directions.

According to [8], the expression for K_ν may be written in the form

$$K_\nu = \frac{c^2 A (\ln 2)^{1/2}}{8\pi\nu^2\Delta\nu'}(n_2 - n_1)\,U\left(\frac{\Delta\nu(\ln 2)^{1/2}}{\Delta\nu'}, \quad \frac{(\nu-\nu_0)(\ln 2)^{1/2}}{\Delta\nu'}\right) \tag{3.2}$$

Here A is the Einstein coefficient for spontaneous emission, c is the speed of light, ν_0 is the transition frequency at the center of the line, $\Delta\nu$ and $\Delta\nu'$ are the collisional and Doppler half-widths of the line, n_2

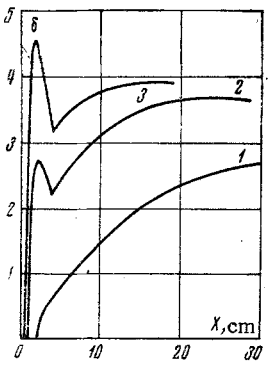

Fig. 1

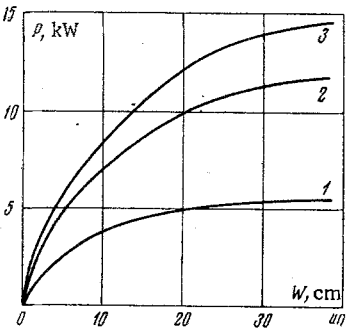

Fig. 2

and n_1 are the population of the upper and lower laser levels respectively, and U is the Voigt function.

The density of molecules $n_V(J)$ at the vibrational-rotational level at equilibrium of the rotational and translation degrees of freedom is

$$n_v(J) = n_v \frac{hcB}{kT}(2J+1)\exp\left[-\frac{hcBJ(J+1)}{kT}\right] \tag{3.3}$$

where n_V is the total population of the vibrational level, J is the rotational quantum number, h is the Planck constant, and B is the rotational constant.

The transition lines of the P-branch ($\Delta J = -1$) corresponding to values of J_0 maximizing (3.3) are the most intense.

For a gas mixture the collisional half-width of the line is given by the relation

$$\Delta\nu = \frac{1}{2\pi}\sum_n N^{(n)}\sigma^{(n)}v^{(n)} \tag{3.4}$$

in which $\sigma^{(n)}$ is the cross section for line broadening by molecules of the n-th species and $v^{(n)}$ is the relative velocity of CO_2 molecules and molecules of the n-th species. The values of $\sigma^{(n)}$ for CO_2, N_2, and He molecules may be found in [9, 10].

The expression for the Doppler half-width of the line has the form

$$\Delta\nu' = \frac{1}{\lambda}\left[\frac{2kT}{m_1}\ln 2\right]^{1/2} \tag{3.5}$$

where λ is the wavelength of the transition in question.

4. One of the most important characteristics to be determined is the radiated power P from the cavity. If W and H are the dimensions of the cavity in the x and y directions, we have

$$P = \int_{x_0}^{x_0+W} I_t H dx \tag{4.1}$$

Here I_t is the output radiation intensity from the cavity:

$$I_t = I^+(L)\,t_2 + I^-(0)\,t_1 \tag{4.2}$$

where $I^+(L)$ and $I^-(0)$ are the intensity values at $z = L, 0$.

The following relations hold in a steady-state radiation field:

$$\frac{I^+(L)}{I^-(L)}r_2 = \frac{I^-(0)}{I^-(0)}r_1 = 1 \tag{4.3}$$

Noting that the number of stimulated transitions per cubic centimeter per unit time can be written in the form

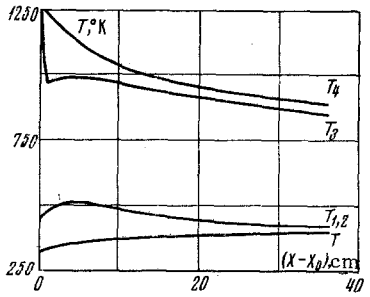

Fig. 3

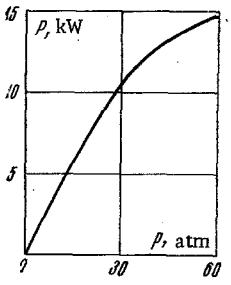

Fig. 4

$$\frac{K_{\nu_0} I}{h\nu} = \frac{1}{h\nu L}\int_0^L K_{\nu_0}(I^+ + I^-)\,dz = \frac{1}{h\nu L}\left[I^+(L) - I^+(0) - I^-(L) + I^-(0)\right] \tag{4.4}$$

and taking (3.1) and (4.3) into account, we obtain

$$I_t = K_{\nu_0} I L\left\{1 - \frac{a_1 + a_2 (r_1 / r_2)^{1/2}}{[1 - (r_1 r_2)^{1/2}]\,[1 + (r_1 / r_2)^{1/2}]}\right\} \tag{4.5}$$

The expression for the radiated power from the cavity has the final form

$$P = LH \int_{x_0}^{x_0 + W} K_{\nu_0} I\left\{1 - \frac{a_1 + a_2 (r_1 / r_2)^{1/2}}{[1 - (r_1 r_2)^{1/2}]\,[1 + (r_1 / r_2)^{1/2}]}\right\} dx \tag{4.6}$$

5. We solved numerically the system (1.4)-(1.10) describing the expansion of the $CO_2 - N_2 - He$ mixture for a flat hyperbolic nozzle with an area ratio described by the formula

$$A / A_* = \sqrt{1 + x^2 / b^2} \tag{5.1}$$

in which $b = h/2 \tan\theta$ (h is the height of the exit slot, and θ is the asymptotic expansion half-angle of the nozzle). The calculations were carried out for a nozzle with h = 0.1 cm and $\theta = 15°$. It follows from our earlier work [4] that for the important case of large densities the motion may be regarded as equilibrium flow up to the critical point. The equilibrium flow equations are easily integrated and the equilibrium value thereby obtained for the volumetric flow of gas through the nozzle. The use of the subcritical equilibrium solution greatly simplifies the problem and shortens the computation time for one set of conditions ("variant").

The distributions of the inverted populations along the nozzle are given for the most significant regimes in Fig. 1, in which the quantity $\delta = [N(00°1) - N(10°0)] \cdot 10^{-15}$ is plotted on the vertical axis. Curves 1, 2, and 3 refer to variants with the following initial conditions:

$P_0 = 15$ atm, $T_0 = 2000°$ K 10% CO_2 — 40% N_2 — 50% He (5.2)
$P_0 = 30$ atm, $T_0 = 2000°$ K, 5% CO_2 — 45% N_2 — 50% He (5.3)
$P_0 = 60$ atm, $T_0 = 2000°$ K, 2.5% CO_2 — 47.5% N_2 — 50% He (5.4)

Note that in all variants the absolute number of CO_2 particles at the entry point is constant.

It follows from Fig. 1 that at higher pressures population inversion takes place earlier along the nozzle due to the increased content of He and, hence, the faster relaxation of the lower laser level.

A difficulty arises in the solution of the system (1.4), (1.5), (1.7), (2.2), (2.5)-(2.8) describing the flow of relaxing gas through the cavity in connection with the fact that for predetermined reflectivities r_1 and r_2 Eq. (2.8), generally speaking, does not holder under arbitrary conditions at the cavity input. It was assumed in [1] for this reason that the populations of the laser levels suffer a discontinuity at the cavity input while all the other variables remain constant.

To remove the discontinuity of the populations at the cavity input we carried out the calculations in the present study for a variable reflectivity $r_2(x)$ specified by a certain function that increases with the length l and then assumes (for $x - x_0 \geq l$) a constant value r_2^+.

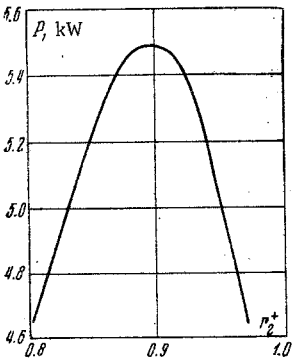

Fig. 5

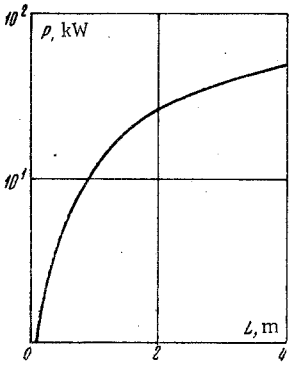

Fig. 6

When the values of T_i, T, u, and ρ are given at the input, Eq. (2.8) is valid only beginning with a certain value r_2^0 at a distance l_0 from the cavity input. For $x - x_0 \leq l_0$ Eq. (2.8) is not used; a steady-state intensity cannot be realized, because in this interval the losses exceed the gain. For $x - x_0 \geq l_0$ the simultaneous solution of Eqs. (2.5)-(2.8) gives the function I(x). It is important to realize the implication of Eqs. (2.5)-(2.8) that I(x) is a discontinuous function at the point $x = x_0 + l_0$, where its value depends on the form of the function $r_2(x)$.

The length l of the variable-reflectivity interval was assumed to be equal in order of magnitude to the diffraction length. We carried out the calculations for several functions $r_2(x)$ and several values of l. For small l in comparison with the characteristic relaxation lengths the mode of specification of the function $r_2(x)$ and the choice of l have virtually no influence on the power value.

The dependence of the radiated power on the cavity length along the x axis is given in Fig. 2. The calculations refer to values of L = 50 cm, $r_1 = 1$, $r_2^+ = 0.9$, and $a_2 = 0.01$. Curves 1, 2, and 3 correspond to conditons (5.2)-(5.4), respectively. The initial increase of the radiated output power is attributed to a rapid exchange of quanta with nitrogen. The subsequent relatively slow growth of the power is due to the slow temperature relaxation of the lower laser level to the translation temperature of the mixture. This fact is evinced by Fig. 3, which shows the behavior of the vibrational and translational temperature in the direction of flow along the cavity for the initial data (5.2). The behavior of the temperature for the other initial data are qualitatively similar.

The dependence of the radiated laser power on the flow stagnation pressure under the condition of an invariant absolute number of CO_2 particles is given in Fig. 4. For sufficiently low pressures the output power essentially increases directly as the pressure. At pressures of ~ 60 atm, however, the power growth slows down, a result that is attributed to rapid collisional relaxation of the upper laser level.

Curves showing the dependence of the radiated output power on r_2^+ and L are given in Figs. 5 and 6. The calculations refer to variant (5.2). We see from Fig. 5 that an optimum reflectivity exists, which is roughly equal to 0.9 for this case. The existence of an optimum reflectivity is explained by the fact that as r_2^+ is increased the radiation intensity in the laser cavity increases on the one hand, while the fraction of emitted radiation decreases on the other. Figure 6 illustrates the dependence of the output power on the length of the laser cavity. Under the given conditions lasing takes place only for a cavity length $L \gtrsim 10$ cm.

The foregoing calculations show that the allowance for collisional relaxation can prove significant in the estimation of radiated output power. A large content of N_2 in the mixture is the most favorable for the generation of large powers. At pressures above ~ 60 atm, however, the power growth slows down due to the rapid collisional relaxation of CO_2. This consideration renders it impractical to further increase the pressure under the stated conditions.

LITERATURE CITED

1. T. A. Cool, "Power and gain characteristics of high-speed flow lasers," J. Appl. Phys., 40, No. 9 (1969).
2. M. Brunne, G. Malaczynski, J. Milenski, and J. Stanco, "Some effects of interaction of monochromatic radiation with a flowing optically active medium, Part II: CW gas-dynamic quantum generator," Bull. Acad. Pol. Sci., 18, No. 3 (1970).

3. M. Brunne, G. Malaczynski, J. Milenski, and J. Stanco, "Some effects of interaction of monochromatic radiation with a flowing optically active medium, Part IV: Modified cw gas-dynamic quantum generator," Bull. Acad. Pol. Sci., 18, No. 6 (1970).
4. N. A. Generalov, G. I. Kozlov, and I. K. Selezneva, "Population inversion of CO_2 molecules in expanded gas flows," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1971).
5. B. F. Gordiets, N. N. Sobolev, and L. A. Shelepin, "Kinetics of physical processes in a CO_2 laser," Zh. Éksp. Teor. Fiz., 53, No. 5 (1967).
6. N. G. Basov, V. G. Mikhailov, A. N. Oraevskii, and V. A. Shcheglov, "Creation of population inversion of molecules in a supersonic flow of a binary gas in Laval nozzles," Zh. Tekh. Fiz., 38, No. 12 (1968).
7. W. W. Rigrod, "Saturation effects in high-gain lasers," J. Appl. Phys., 36, No. 8 (1965).
8. C. K. N. Patel, "Continuous-wave laser action on vibrational-rotational transitions of CO_2," Phys. Rev., 136, No. 5A (1964).
9. A. D. Devir and U. P. Oppenheim, "Linewidth determination in the 9.4 μ and 10.4 μ bands of CO_2 using a CO_2 laser," Appl. Opt., 8, No. 10 (1969).
10. R. R. Patty, E. R. Manring, and J. A. Gardner, "Determination of self-broadening coefficients of CO_2 using CO_2 laser radiation at 10.6 μ," Appl. Opt., 7, No. 11 (1968).